Considere o seguinte sistema não linear:

\[ x - y^2 + 2y = 0 \]
\[ 2x + y^2 = 6 \]

```maple
restart;

with(linalg):
with(plots):

> g1 := (x, y) -> (x - y*y + 2*y);
> g1 := (x, y) -> x - y^2 + 2*y

> g2 := (x, y) -> (2*x + y^2 - 6);
> g2 := (x, y) -> 2*x + y^2 - 6

> plot3d({x - y*y + 2*y, 2*x + y^2 - 6}, x = 0..1, y = 0..1);
> plot3d({x - y*y + 2*y, 2*x + y^2 - 6}, x = -1..1, y = -1..1);
```
plot3d(x-y^2+2*y, 2*x+y^2-6, x=-2..1, y=-2..1);

implicitplot(x-y^2+2*y, 2*x+y^2-6, x=-3..3, y=-3..3);

plot3d(x-y^2+2*y, 2*x+y^2-6, x=-2..1, y=-2..1);

implicitplot(x-y^2+2*y, 2*x+y^2-6, x=-3..3, y=-3..3);
Note pelo gráfico que temos duas soluções aproximadas no intervalo [-3,3]x[-3,3]. Uma solução X_1=(x_1, y_1), onde x_1 está em [0,1] e y_1 está em [2,3].

Uma segunda solução X_2=(x_2, y_2), onde x_2 está em [2,3] e y_2 está em [-1,0]. Vamos aplicar o método de Newton para obter as duas soluções.

\[ X^{[n+1]} = X^n - \frac{F[X^n]}{J[X^n]} \]

DENOTANDO

\[ J[X^n] \] \( \ast \) \( Y^n \) = - \( F[X^n] \) então \( X^{[n+1]} = X^{[n]} + Y^{[n]} \), para \( n = 0,1,2,... \)

\[ \text{Jaco} := (x,y) \rightarrow \text{array}(1..2,1..2,[[1,-2*y+2],[2,2*y]]); \]

\[ F:=(x,y) \rightarrow \text{array}(1..2,[[x-y^2+2*y],[2*x+y^2-6]]); \]

OBTENDO A PRIMEIRA SOLUÇÃO DO SISTEMA NÃO LINEAR. \( X=(x_1, y_1) \)

TOMAREMOS COMO APROXIMAÇÃO INICIAL O VALOR DE \( X^0 = [0.5, 2.5] \)

\[ x_0:=0.5; \]
\[ y_0:=2.5; \]
\[ X_0:=[x_0, y_0]; \]
\[ J_0:=\text{evalf} (\text{Jaco}(x_0,y_0)); \]
\[ F_0:=\text{evalf} (F(x_0,y_0)); \]
\[ b:=-F_0; \]
\[ Y_0:=\text{multiply}(\text{inverse}(J_0),b); \]
\[ X_1:=\text{evalf} (\text{evalm}(X_0+Y_0),5); \]
\( x_1 := 0.5; \)
\( y_1 := 2.25; \)
\( X_1 := [x_1, y_1]; \)
\( J_1 := \text{evalf}(\text{Jaco}(x_1, y_1)); \)
\( J_1 := \begin{bmatrix} 1 & -2.5 \end{bmatrix} \)
\( F_1 := \text{evalf}(F(x_1, y_1)); \)
\( b := -F_1; \)
\( Y_1 := \text{multiply}(\text{inverse}(J_1), b); \)
\( X_2 := \text{evalf}(\text{evalm}(X_1 + Y_1), 5); \)
\( x_2 := 0.51316; \)
\( y_2 := 2.2303; \)
\( X_2 := [x_2, y_2]; \)
\( J_2 := \text{evalf}(\text{Jaco}(x_2, y_2)); \)
\( J_2 := \begin{bmatrix} 1 & -2.4606 \end{bmatrix} \)
\( F_2 := \text{evalf}(F(x_2, y_2)); \)
\( b := -F_2; \)
\( Y_2 := \text{multiply}(\text{inverse}(J_2), b); \)
\( X_3 := \text{evalf}(\text{evalm}(X_2 + Y_2), 5); \)
\( x_3 := 0.51324; \)
\( y_3 := 2.2301; \)
\( X_3 := [x_3, y_3]; \)
\( X_3 := [0.51324, 2.2301] \)

Solução aproximada do sistema não linear:
\( X_3 := [0.51324, 2.2301] \)

como erro menor que 1.6x10^{-4}.
considere como solução inicial o seguinte valor $X_0=(2.5; -0.5)$

> $x_0 := 2.5$
> $y_0 := -0.5$
> $X_0 := [x_0, y_0]$
> $J_0 := \text{evalf}(\text{Jaco}(x_0, y_0))$
> $J_0 := \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}$
> $F_0 := \text{evalf}(\text{F}(x_0, y_0))$
> $b := -F_0$
> $Y_0 := \text{multiply}(\text{inverse}(J_0), b)$
> $Y_0 := \begin{bmatrix} 0.1428571429 \\ -0.4642857143 \end{bmatrix}$
> $X_1 := \text{evalf}(\text{evalm}(X_0 + Y_0), 5)$
> $X_1 := [2.6429, -0.96429]$
\[ J_2 := \begin{bmatrix} 1 & 3.79640 \\ 2 & -1.79640 \end{bmatrix} \]

\[ F_2 := \text{evalf}(F(x_2, y_2)); \]
\[ F_2 := \begin{bmatrix} \cdot004363240 \\ \cdot001394125165 \end{bmatrix} \]

\[ b := - F_2; \]
\[ b := \begin{bmatrix} \cdot004363240 \\ \cdot004363240 \end{bmatrix} \]

\[ Y_2 := \text{multiply}(\text{inverse}(J_2), b); \]
\[ Y_2 := \begin{bmatrix} 2.5979 \\ -0.89681 \end{bmatrix} \]

\[ X_3 := \text{evalf}(\text{evalm}(X_2 + Y_2), 5); \]
\[ X_3 := \begin{bmatrix} 2.5979 \\ -0.89681 \end{bmatrix} \]

Solução aproximada do sistema não linear: \[ X_3 = \begin{bmatrix} 2.5979 \\ -0.89681 \end{bmatrix} \]
com erro menor que \(1.4 \times 10^{-3}\)

**EXEMPLO 2**

SISTEMA NÃO LINEAR
\[ x \cdot \sin(x) \cdot \cos(y) = 0 \]
\[ x \cdot \cos(x) \cdot \cos(y) = 0 \]
\[ x \cdot \sin(y) + z = 0 \]

APROXIMAÇÃO INICIAL: \[ X^0(x_0, y_0, z_0) = (-\pi/2, \pi, 0) \]

\[ \text{Jaco} := (x, y, z) \rightarrow \text{array}(1..3, 1..3, \begin{bmatrix} \sin(x) \cdot \cos(x) \cdot \cos(y), -x \cdot \sin(x) \cdot \sin(y), 0 \\ \cos(x) - x \cdot \sin(x) \cdot \sin(y), 0 \end{bmatrix}); \]

\[ \text{Jaco} := (x, y, z) \rightarrow \begin{bmatrix} \sin(x) \cdot \cos(x) \cdot \cos(y), -x \cdot \sin(x) \cdot \sin(y), 0 \\ \cos(x) - x \cdot \sin(x) \cdot \sin(y), 0 \end{bmatrix}; \]

\[ X_0 := [x_0], [y_0], [z_0]; \]

\[ f1 := (x, y, z) \rightarrow (x \cdot \sin(x) \cdot \cos(y)); \]
\[ f2 := (x, y, z) \rightarrow (x \cdot \cos(x) \cdot \cos(y)); \]
\[ f3 := (x, y, z) \rightarrow (x \cdot \sin(y) + z); \]

\[ F := (x, y, z) \rightarrow \text{array}(1..3, \begin{bmatrix} x \cdot \sin(x) \cdot \cos(y), x \cdot \cos(x) \cdot \cos(y), x \cdot \sin(y) + z \end{bmatrix}); \]

\[ X_0 := [x_0], [y_0], [z_0]; \]
EXEMPLO 3

SISTEMA NÃO LINEAR

x*\sin(x)\cos(y)-z =0
x*\cos(x)\cos(y)=0
x*\sin(y)+z =0

APROXIMAÇÃO INICIAL X^0=(x_0,y_0, z_0) = (-\frac{\pi}{2}, \pi, 0 )

MATRIZ JACOBIANA É SINGULAR
\[ J_0 := \begin{bmatrix} 1. & 0 & -1. \\ 1.570796327 & 0 & 0 \\ 0 & 1.570796327 & 1. \end{bmatrix} \]

\[ F_0 := \begin{bmatrix} [-1.570796327], [0], [0] \end{bmatrix} \]

\[ c := \text{det}(J_0); \]
\[ c := -2.467401101 \]

\[ F_0 := \text{evalf}(F(x_0, y_0, z_0)); \]

\[ b := -F_0; \]

\[ Y_0 := \text{evalm}(\text{multiply}(\text{inverse}(J_0), b)); \]

\[ Y_0 := \begin{bmatrix} 0 \\ 1.000000000 \\ -1.570796327 \end{bmatrix} \]

\[ X_1 := \text{evalf}(\text{evalm}(X_0 + Y_0), 5); \]

\[ x_1 := -1.5708; \]

\[ y_1 := -2.1416; \]

\[ z_1 := -1.5708; \]

\[ J_1 := \text{evalf}(\text{Jaco}(x_1, y_1, z_1)); \]

\[ J_1 := \begin{bmatrix} .5403053701 & 1.321776388 & -1. \\ .8487185569 & .4855155773 \times 10^5 & 0 \\ -.8414670155 & .8487165723 & 1. \end{bmatrix} \]

\[ F_1 := \text{evalf}(F(x_1, y_1, z_1)); \]

\[ b := -F_1; \]

\[ Y_1 := \text{evalm}(\text{multiply}(\text{inverse}(J_1), b)); \]

\[ Y_1 := \begin{bmatrix} .4919993625 \times 10^5 \\ -.2179497205 \\ .4340052918 \end{bmatrix} \]

\[ X_2 := \text{evalf}(\text{evalm}(X_1 + Y_1), 5); \]

\[ x_2 := 2.6429; \]

\[ y_2 := -1.1822; \]

\[ z_2 := -1.1368; \]

\[ J_2 := \text{evalf}(\text{Jaco}(x_2, y_2, z_2)); \]

\[ J_2 := \begin{bmatrix} 2.6429 \\ -1.1822 \\ -1.1368 \end{bmatrix} \]
> J_2 := 
\[
\begin{bmatrix}
-0.6981952038 & 1.169795892 & -1. \\
-0.8116762518 & -2.147966610 & 0 \\
-0.9254418088 & 1.001367776 & 1. \\
\end{bmatrix}
\]

> F_2 := evalf(F(x_2, y_2, z_2));

\[
F_2 := [\begin{bmatrix}
1.615732001 \\
-0.8794097794 \\
-3.582650156 \\
\end{bmatrix}
\]

> b := - F_2

\[
b := -F_2
\]

> Y_2 := evalm(multiply(inverse(J_2), b));

\[
Y_2 := \begin{bmatrix}
-1.168466797 \\
-0.0321266498 \\
2.469131537 \\
\end{bmatrix}
\]

> X_3 := evalf(evalm(X_2 + Y_2), 5);

\[
X_3 := \begin{bmatrix}
1.4744 \\
-1.1501 \\
1.3323 \\
\end{bmatrix}
\]

> x_3 := 1.4744;

\[
x_3 := 1.4744
\]

> y_3 := -1.1501;

\[
y_3 := -1.1501
\]

> z_3 := 1.3323;

\[
z_3 := 1.3323
\]

> J_3 := evalf(Jaco(x_3, y_3, z_3));

\[
J_3 := \begin{bmatrix}
0.4644543365 & 1.339591276 & -1. \\
-0.5600369038 & 0.1295331454 & 0 \\
-0.9128047844 & 0.6021393018 & 1. \\
\end{bmatrix}
\]

> F_3 := evalf(F(x_3, y_3, z_3));

\[
F_3 := [\begin{bmatrix}
-0.7329561474 \\
0.05795416542 \\
-0.013539374 \\
\end{bmatrix}
\]

> b := - F_3

\[
b := -F_3
\]

> Y_3 := evalm(multiply(inverse(J_3), b));

\[
Y_3 := \begin{bmatrix}
0.2032586908 \\
0.4313814985 \\
-0.06067687489 \\
\end{bmatrix}
\]

> X_4 := evalf(evalm(X_3 + Y_3), 5);

\[
X_4 := \begin{bmatrix}
1.6777 \\
-0.71872 \\
1.2716 \\
\end{bmatrix}
\]

> x_4 := 1.6777;

\[
x_4 := 1.6777
\]

> y_4 := -0.71872;

\[
y_4 := -0.71872
\]

> z_4 := 1.2716;

\[
z_4 := 1.2716
\]
\[ J_4 := \text{evalf}(\text{Jaco}(x_4, y_4, z_4)); \]
\[
\begin{bmatrix}
0.6136200595 & 1.098328199 & -1. \\
-1.335818666 & -1.178646626 & 0 \\
-0.6584218207 & 1.262719438 & 1.
\end{bmatrix}
\]
\[ J_4 \]
\[
\begin{bmatrix}
0.6136200595 & 1.098328199 & -1. \\
-1.335818666 & -1.178646626 & 0 \\
-0.6584218207 & 1.262719438 & 1.
\end{bmatrix}
\]

\[ F_4 := \text{evalf}(F(x_4, y_4, z_4)); \]
\[ F_4 := \begin{bmatrix} -0.16089122, [-0.1347323744], [0.1669657111] \end{bmatrix} \]
\[ b := -F_4 \]

\[ Y_4 := \text{evalm}(\text{multiply}(\text{inverse}(J_4), \text{b})); \]
\[ Y_4 := \begin{bmatrix} -0.09506374726 \\
0.06570626101 \\
-0.1465891836 \end{bmatrix} \]

\[ X_5 := \text{evalf}(\text{evalm}(X_4 + Y_4), 5); \]
\[ X_5 := \begin{bmatrix} 1.5826 \\
-0.78443 \\
1.1250 \end{bmatrix} \]

\[ x_5 := 1.5826; \]
\[ y_5 := -0.78443; \]
\[ z_5 := 1.1250; \]

\[ J_5 := \text{evalf}(\text{Jaco}(x_5, y_5, z_5)); \]
\[ J_5 := \begin{bmatrix} 0.6945201594 & 1.117905346 & -1. \\
-1.128426415 & -0.01361960223 & 0 \\
-0.7064218550 & 1.120150107 & 1. \end{bmatrix} \]

\[ F_5 := \text{evalf}(F(x_5, y_5, z_5)); \]
\[ F_5 := \begin{bmatrix} -0.004927925 \\
-0.01322157878 \\
0.007016772 \end{bmatrix} \]
\[ b := -F_5 \]

\[ Y_5 := \text{evalm}(\text{multiply}(\text{inverse}(J_5), \text{b})); \]
\[ Y_5 := \begin{bmatrix} -0.01170518605 \\
0.009995377906 \\
-0.01417037455 \end{bmatrix} \]

\[ X_6 := \text{evalf}(\text{evalm}(X_5 + Y_5), 5); \]
\[ X_6 := \begin{bmatrix} 1.5709 \\
-0.78543 \\
1.1108 \end{bmatrix} \]

\[ x_6 := 1.5709; \]
\[ y_6 := -0.78543; \]
\[ z_6 := 1.1108; \]
J_6 := evalf(Jaco(x_6, y_6, z_6));
J_6 :=
\[
\begin{pmatrix}
0.7069691093 & 1.110829400 & -1.\\
-1.110831978 & -0.0011151632446 & 0 \\
-0.7071292927 & 1.110758678 & 1.
\end{pmatrix}
\]

F_6 := evalf(F(x_6, y_6, z_6));
F_6 := \begin{bmatrix}
0.7069691093 & 1.110829400 & -1.
-1.110831978 & -0.0011151632446 & 0 \\
-0.7071292927 & 1.110758678 & 1.
\end{bmatrix}

b := -F_6

Y_6 := evalm(multiply(inverse(J_6), b));
Y_6 :=
\[
\begin{pmatrix}
-0.0001036696641 \\
0.00003183146081 \\
-0.0007925892750
\end{pmatrix}
\]

X_7 := evalf(evalm(X_6 + Y_6), 5);
X_7 :=
\[
\begin{pmatrix}
1.5708 \\
-0.78540 \\
1.1107
\end{pmatrix}
\]

Após iterações obtemos como solução aproximada o vetor X=(1.5708, -0.78540, 1.1107)
O erro nesse caso 1x10^-4