CÁLCULO NUMÉRICO - Gabarito Lista No. 6

1. (a)

m	4	6
Trapezios	4.6950759	4.6815792
Simpson	4.670873	4.6707894

$$E_{TR} \leq \frac{(b-a)h^2}{12} \max_{a\leq x\leq b} |f''(x)| = \begin{cases} \frac{1(1/4)^2 e^2}{12} = 0.0384847 \\ \frac{1(1/6)^2 e^2}{12} = 0.0171043 \end{cases}$$

$$E_{SR} \leq \frac{(b-a)h^4}{180} \max_{a\leq x\leq b} |f^{(4)}(x)| = \begin{cases} \frac{1(1/4)^4 e^2}{180} = 0.000160353 \\ \frac{1(1/6)^4 e^2}{180} = 0.0000316746 \end{cases}$$

(b)

m	4	6
Trapezios	4.6550925	4.6614884
Simpson	4.6662207	4.6665612

$$E_{TR} \leq \frac{(b-a)h^2}{12} \max_{a\leq x\leq b} |f''(x)| = \begin{cases} \frac{3(3/4)^2(1^{-3/2})/4}{12} = 0.0351563 \\ \frac{3(3/6)^2(1^{-3/2})/4}{12} = 0.015625 \end{cases}$$

$$E_{SR} \leq \frac{(b-a)h^4}{180} \max_{a\leq x\leq b} |f^{(4)}(x)| = \begin{cases} \frac{3(3/4)^4 15(1^{-7/2})/16}{180} = 0.00494385 \\ \frac{3(3/6)^4 15(1^{-7/2})/16}{180} = 0.000976563 \end{cases}$$

(c)

m	4	6
Trapezios	4.7683868	4.7077771
Simpson	4.6763744	4.6614894

$$E_{TR} \leq \frac{(b-a)h^2}{12} \max_{a\leq x\leq b} |f''(x)| = \begin{cases} \frac{12(12/4)^2 3(2^{-5/2})/4}{12} = 1.19324 \\ \frac{12(12/6)^2 3(2^{-5/2})/4}{12} = 0.53033 \end{cases}$$

$$E_{SR} \leq \frac{(b-a)h^4}{180} \max_{a\leq x\leq b} |f^{(4)}(x)| = \begin{cases} \frac{12(12/4)^4 105(2^{-9/2})/16}{180} = 1.56613 \\ \frac{12(12/6)^4 105(2^{-9/2})/16}{180} = 0.309359 \end{cases}$$

2. O erro é zero.

3. Temos que

$$E_{SR} \leq \frac{(b-a)h^4}{180} \max_{a\leq x\leq b} |f^{(4)}(x)| = \frac{(\pi/2)h^4}{180} \max_{0\leq x\leq \pi/2} |\cos(x)|$$

$$E_{SR} \leq \frac{\pi h^4}{360} < \epsilon = 10^{-3} \quad \Rightarrow \quad h < \sqrt[4]{\frac{360 \cdot 10^{-3}}{\pi}} = 0.581819.$$

4. Como o número de subíntervalos é ímpar não podemos usar a regra de Simpson. Então, temos que combina-la com a regra dos trapézios. O melhor é usar a regra dos trapézios no intervalo $[0.0, 0.2]$ (onde a função varia mais devagar) e a de Simpson em $[0.2, 1.0]$. Obtemos que

$$I \approx \frac{h}{2}\{f(x_0) + f(x_1)\} + \frac{h}{3}\sum_{i=1}^{2}\{f(x_{2i-1}) + 4f(x_{2i}) + f(x_{2i+1})\}$$
$$I \approx \frac{0.2}{2}\{1.0 + 1.2408\} + \frac{0.2}{3}\{1.2408 + 4 \times 1.5735 + 2 \times 2.0333 + 4 \times 2.6965 + 3.7183\}$$
$$I \approx 1.96446$$

8. Temos que

$$M_4 = \max_{0\le x\le 1}\left|\frac{d^4}{dx^4}\left(\frac{1}{1+x^2}\right)\right| = \max_{0\le x\le 1}\left|\frac{24(1-10x^2+5x^4)}{(1+x^2)^5}\right| = 24$$
$$E_{SR} \le \frac{(b-a)M_4h^4}{180} = \frac{24(1/m)^4}{180} < 0.25\cdot 10^{-3} \quad \Rightarrow$$
$$m > \sqrt[4]{\frac{24}{180\cdot 0.25\cdot 10^{-3}}} = 4.80562 \quad \Rightarrow m = 6.$$

Então

$$\pi \approx \frac{4h}{3}\sum_{i=0}^{m/2-1}\{f(x_{2i}) + 4f(x_{2i+1}) + f(x_{2i+2})\}$$
$$\approx \frac{4\cdot\frac{1}{6}}{3}\left(\frac{1}{1+(\frac{0}{6})^2} + \frac{4}{1+(\frac{1}{6})^2} + \frac{2}{1+(\frac{2}{6})^2} + \frac{4}{1+(\frac{3}{6})^2} + \frac{2}{1+(\frac{4}{6})^2} + \frac{4}{1+(\frac{5}{6})^2} + \frac{1}{1+(\frac{6}{6})^2}\right)$$
$$\approx 3.141591781.$$