CÁLCULO NUMÉRICO

Lista No. 3 – Gabarito

1. Solução exata: $(x_1 = 111/44; x_2 = 1/44; x_3 = 3/44; x_4 = 75/44)$

2. *a)* $(x_1 = 0.00; x_2 = 10.0; x_3 = 0.143)$
b) $(x_1 = 69.0; x_2 = -349; x_3 = 321)$
c) $(x_1 = 0.183; x_2 = 0.0103; x_3 = -0.0200; x_4 = -1.12)$

3. *a)* $(x_1 = 0.00; x_2 = 10.0; x_3 = 0.143)$
b) $(x_1 = 73.0; x_2 = -373; x_3 = 344)$
c) $(x_1 = 0.178; x_2 = 0.0127; x_3 = -0.0204; x_4 = -1.16)$

4. *a)* $(x_1 = 0.00; x_2 = 10.0; x_3 = 0.143)$
b) $(x_1 = 73.0; x_2 = -370; x_3 = 342)$
c) $(x_1 = 0.179; x_2 = 0.0127; x_3 = -0.0203; x_4 = -1.15)$

5. *a)*

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 7.5 \\ 0 & 0 & -4 \end{bmatrix}$$

b)

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & -1.5 & 3 \\ 0 & 0.75 & 3.5 \\ 0 & 0 & -8 \end{bmatrix}$$

c)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.84923 & 1 & 0 & 0 \\ -0.459652 & -0.250117 & 1 & 0 \\ 2.7871 & -0.307956 & -5.35223 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2.17556 & 4.0231 & -2.1732 & 5.1967 \\ 0 & 13.4396 & -4.01874 & 10.8072 \\ 0 & 0 & -0.892969 & 5.09173 \\ 0 & 0 & 0 & 12.0359 \end{bmatrix}$$

6. *a)* $(x_1 = 0.5; x_2 = 0; x_3 = 0)$
b) $(x_1 = 0.875; x_2 = 0.708333; x_3 = -0.0625)$
c) $(x_1 = 0.0743235; x_2 = -0.742971; x_3 = -2.72854; x_4 = -0.500761)$

7. *a)* $\det A = \det A^{(n-1)} = a_{11}^{(n-1)} a_{22}^{(n-1)} \cdots a_{nn}^{(n-1)} = a_{11}^{(0)} a_{22}^{(1)} \cdots a_{nn}^{(n-1)}$

10. Usamos sempre a aproximação inicial $x^{(0)} = \mathbf{0}$.

a)

$$\alpha_1 = \alpha_2 = \alpha_3 = 0.2 \quad \Rightarrow \quad \boxed{\alpha = 0.2 < 1}$$
$$\beta_1 = 0.2,\ \beta_2 = 0.12,\ \beta_3 = 0.032 \quad \Rightarrow \quad \boxed{\beta = 0.2 < 1}$$

$$\text{Gauss-Jacobi} \quad x^{(7)} = \begin{pmatrix} 1.0000128 \\ 1.0000128 \\ 1.0000128 \end{pmatrix}, \quad \text{Gauss-Seidel} \quad x^{(5)} = \begin{pmatrix} 1.0000002 \\ 0.9999999 \\ 1.0000000 \end{pmatrix}$$

b)

$$\alpha_1 = \alpha_4 = 0.25,\ \alpha_2 = \alpha_3 = 0.5 \quad \Rightarrow \quad \boxed{\alpha = 0.5 < 1}$$
$$\beta_1 = 0.25,\ \beta_2 = 0.3125,\ \beta_3 = 0.328125,\ \beta_4 = 0.0820313 \quad \Rightarrow \quad \boxed{\beta = 0.2 < 1}$$

$$\text{Gauss-Jacobi} \quad x^{(10)} = \begin{pmatrix} 0.3636007 \\ 0.4544878 \\ 0.4544878 \\ 0.3636007 \end{pmatrix}, \quad \text{Gauss-Seidel} \quad x^{(7)} = \begin{pmatrix} 0.3636286 \\ 0.4545404 \\ 0.4545434 \\ 0.3636358 \end{pmatrix}$$

11. Inicialmente obtemos:

$$x^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \cdots, x^{(5)} = \begin{pmatrix} -7909.1563 \\ 23729.469 \end{pmatrix}, \cdots, x^{(20)} = 10^{17} \begin{pmatrix} -1.0570706 \\ 3.1712119 \end{pmatrix}, \cdots;$$

o método **não converge**.

Permutando as equações obtemos a solução aproximada em 6 iterações:

$$x^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \cdots, x^{(6)} = \begin{pmatrix} 0.9999860 \\ -0.9999944 \end{pmatrix}.$$

12. *a)*

$$k > 0, \quad \beta_1 = \frac{4}{k}, \quad \beta_2 = \frac{5}{k}, \quad \beta_3 = \frac{34}{7k} \quad \Rightarrow \quad \beta = \frac{5}{k} < 1 \quad \Rightarrow \quad \boxed{k > 5}.$$

b) $k = 6$.

$$x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad x^{(1)} = \begin{pmatrix} 0.1666667 \\ 0.1666667 \\ 0.2619048 \end{pmatrix}, \quad x^{(2)} = \begin{pmatrix} 0.0396825 \\ 0.25 \\ 0.2086168 \end{pmatrix}.$$